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PHOTOGRAPHS OF THE CORONA

TAKEN DURING THE

TOTAL ECLIPSE OF THE SUN,

JANUARY 1, 1889.

STRUCTURE OF THE CORONA.

BY

DAVID P. TODD, PH.D.,

Director Amherst College Observatory.

CITY OF WASHINGTON:
PUBLISHED BY THE SMITHSONIAN INSTITUTION.
1889.

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INTRODUCTORY NOTE.

The accompanying plates have been prepared from positive copies on glass of photographs of the total eclipse of the sun of January 1, 1889, kindly presented to the Smithsonian Institution by Professors Pickering, Holden, and Payne, Captain Floyd, General Irish, and Mr. Burckhalter. The copies have, for the sake of comparison, been reduced to a uniform diameter by Mr. Smillie, the photographer of the Institution, and a descriptive note with remarks on the structure of the corona has been added at my request by Professor Todd.

It is not intended to include these plates in the Contributions to Knowledge, but a limited number of carefully prepared prints will be distributed to astronomers and others specially interested in solar physics.

S. P. LANGLEY,
Secretary.

SMITHSONIAN INSTITUTION,
October 1, 1889.

ON THE STRUCTURE OF THE CORONA

AS INDICATED BY THE PHOTOGRAPHS TAKEN 1889, JANUARY 1.

By Professor David P. Todd.

On occasion of the eclipse of this date clear skies were everywhere prevalent. A great variety of photographic apparatus was in the field and a rich harvest of pictures was gathered.

At the request of Professor Langley, glass positives of all the better photographs were forwarded to the Smithsonian Institution for comparison and preservation. These positives being contact prints, there was, of course, great diversity of scale, and the first step was to enlarge or reduce the photographs to conformity with an arbitrary unit. For the unit of the moon's diameter Professor Langley chose 25 millimeters, and the necessary negatives of this size were prepared by Mr. Smillie, the photographer of the Institution. Prints from these secondary negatives form Plate I. Of course the results would have been better had all the original negatives been available. Captain Floyd alone transmitted an original negative.

Also a series of lantern positives was prepared, and the photographs were exhibited to the National Academy of Sciences on the 17th April, 1889. Certain conclusions were drawn from the collation of the photographs, and suggestions made for the observation of future eclipses. In the main these follow.

In the subjoined table are presented all the important circumstances and conditions pertaining to these nine photographs of the corona. The top and bottom of each print are north and south, and the right and left are west and east, respectively. The sun's axis is inclined at an angle of between one and two degrees with the north and south line (position angle $= + 1^{\circ}.4$), and the vertex or highest point of the sun and the trace of the ecliptic are shown with sufficient accuracy for each figure by the diagram in Plate II.

The excellence of Nos. 1 and 2 indicates the desirability of photographing the corona with reflectors in the future.

In No. 3 the poor definition appears to be due partly to the lack of exact focal adjustment and partly to the deficient clock-work, a temporary apparatus having been devised for turning the polar axis by hand.

That a glass of less than two inches aperture and unadapted to photography should have produced a photograph (No. 4) comparable with that obtained (No. 5) by a 13-inch objective specially corrected for the photographic rays is a matter requiring minute investigation, and is suggestive as to eclipse outfits in the future. There appear to be other effects than those due to difference of aperture merely. Both these pictures are shown in Plate II of the size of the original negatives.

In No. 8 the effect of the dry-plate granules is conspicuously brought out by excessive enlargement. To a slight extent this is apparent in No. 9 also.

In commenting on this collection of photographs before the Academy, I ventured the following observations:

(I) The axis of symmetry of the corona does not coincide with the axis of revolution of the sun as determined from the solar spots. The corona appears to be at least a triple phenomenon* made up of—

(*a*) The polar rays, seen most prominently about the poles, but probably extending into the equatorial regions, and not there seen because projected upon the filaments which have their proper origin there.

(*b*) The inner equatorial corona, the lower regions of which bear some resemblance to an outer solar atmosphere, and have perhaps a closer connection with truly solar phenomena than any other part.

(*c*) The outer equatorial corona, consisting of the long streamers, for the most part visible only to the naked eye, and having perhaps no necessary connection with the sun.

(II) The polar corona consists of rays, straight or nearly so, and radial from neither the sun's centre nor the sun's poles. Rather they seem to radiate from areas the centres of which are adjacent to the sun's poles; and the law of their inclination to the polar axis of the corona appears to be susceptible of precise empirical determination.

A few of these rays appear to be double nearly their whole length. For the most part, if not entirely, the rays or beams have parallel sides. They can

* Note in this connection Young and Huggins on the compound spectrum of the corona, consisting of three superposed spectra. Silliman's Journal, vol. 102, p. 53, and Proc. Royal Society, 239, 1885, page 121.

be subdivided with increase of magnifying power, and are sharply defined like the lines of the sun's spectrum. Between them the dark rifts sometimes extend quite to the disk of the moon.

Occasionally a ray appears to have a slight curvature, in general from, but often toward, the solar axis, and this may be due to optical or to photographic illusion, or both.

(III) The inner equatorial corona emits a large percentage of the total light of the corona. Photographs taken with clock-work show great detail in this region, though the streamers are not generally so sharply defined as about the poles. Many of these streamers appear to have a real curvature.

Four large prominences are visible, two on each side of the sun and at about 35° of solar latitude, as if to suggest some connection between the protuberances and the corona.

(IV) The equatorial streamers of the corona are for the most part lost by the operation of enlargement. These streamers are very slightly curved, while they are convergent on the east side of the sun and divergent on the west.

On the latter side and about 1° from the sun's centre there is a suggestion of wider divergence, as if there were electric repulsion between adjacent streamers, as Huggins' theory would imply. The photographic evidence as to the existence of a meteoritic ring or equatorial envelope surrounding the sun is inconclusive.

The fact of chief importance established appears to be the periodicity of the outer corona in a cycle probably of equal duration with that of the solar spots. A comparison of the corona of 1889 with those of 1867 (Grosch), 1868 (Bullock), and 1878 (Langley, Newcomb, and others) is sufficient to establish this periodicity beyond reasonable doubt. The epoch of greatest extension of the equatorial corona appears to coincide very nearly with the epoch of minimum sun spots.

(V) No rapid change in the structure of the corona can safely be inferred from a comparison of photographs taken at the different stations. The differences are slight, if any, and may well be due to differences of objectives, plates, and development.

The time-difference between the photographs at Bartlett Springs and at Willows is about one minute. The only safe inference appears to be that, while the corona may change from hour to hour, there is no present indication of change from minute to minute.

In order to investigate this question fully in the future, it will be necessary to make two like series of exposures at widely separate stations, and combine each series into an accurate representation of the entire corona, if possible, by means of composite photography.

(VI) A few suggestions for the coming eclipse bearing on coronal structure are pertinent. They are the more important as the decade of the nineties contains only two eclipses likely to be well observed—1893, April 16, in Brazil and West Africa, and 1898, January 22, in East Africa and the west of India:

(1) For the minute study of the detailed structure of the corona it is absolutely essential that the photographic telescope be equatorially mounted and driven by perfect clock-work.

(2) These photographs should be taken on a scale as large as convenient in order to avoid effects of the granulation of the dry-plate films.

(3) Means must be provided for the most accurate orientation of the polar streamers. If a plumb-line or the parallel cannot be photographed on the plate, exact orientation should be accomplished by optical measurement of the position angle of a suitable prominence.

(4) Attention should be directed to photographing the outer coronal streamers by the most delicate apparatus available.

(5) In view of differences in the photographic correction of objectives, pairs of reflecting telescopes should be used, widely distant on the earth's surface, and with identical plates. In view of the greater absorption of the H K rays by silver on glass, the reflector would preferably be made of speculum metal.

If dioptric instruments have to be used, the actinic focus for the coronal rays should be experimentally determined with the greatest precison.

(6) Comparisons of different photographs and generalizations upon the coronal structure are not worth attempting unless the precise relation of the solar to the lunar centre is known. It is therefore essential that the exact time of the middle of each exposure be known, together with the longitude and latitude of the station within 15″.

PHOTOGRAPHS OF THE TOTAL SOLAR ECLIPSE, January 1, 1889.

Number.	Name.	Place in California.	Longitude west of Greenwich.	Latitude.	Elevation above sea-level.	Approximate distance from line of central eclipse.	Duration of totality.	Telescope or camera. Clear aperture.	Telescope or camera. Focal length.	Image of moon. Original size.	Image of moon. Enlarged diameters.	Maker of telescope or camera.	Maker of photographic plate.	Sensitometer number.¶	Exposure. Began. Second of totality.	Exposure. Length.	Driving-clock.	Position-angle of sun's axis from north point toward east.
			h. m. s.	° ′ ″	feet.	miles.	m. s.	in.	in.	in.					th.	s.		°
1	C. Burckhalter	Cloverdale	8 11 50	N. 38 47 40	315	22 S.	1 44	* $10\frac{1}{2}$	84	0.72	1.2	Brashear	Passavant	22 ?	35	1	None	342.1
2	C. Burckhalter	Cloverdale	8 11 50	N. 38 47 40	315	22 S.	1 44	* $10\frac{1}{2}$	84	0.72	1.2	Brashear	Passavant	15 ?	77	5	None	342.1
3	R. S. Floyd	Clear Lake	8 11 4	N. 39 2 --	---	12 S.	1 57	† 5	65.6	0.68	1.4	Clark & Sons	Seed	26	10	3	None	341.9
4	E. E. Barnard	Bartlett Springs	8 11 --	N. 39 17 --	---	3 N.	1 59	§ $3\frac{1}{2}$	49	0.48	2.0	Clark & Sons	Seed	26	107	4.5	Clock	341.6
5	W. H. Pickering	Willows	8 8 40	N. 39 32 --	250	9 N.	1 58	† 13	196	1.84	0.5	Clark & Sons	Carbutt (A.)	12	38	10	Clock	341.6
6	W. H. Pickering	Willows	8 8 40	N. 39 32 --	250	9 N.	1 58	8	44	0.42	2.4	Voigtländer ‖	Carbutt (A.)	12	58	10	Clock	341.6
7	W. W. Payne	Chico	8 7 28	N. 39 43 56	193	13 N.	1 55	* 6	--	0.56	1.8	Brashear	Seed	22	80	10	None	341.3
8	W. W. Payne	Chico	8 7 28	N. 39 43 56	193	13 N.	1 55	$2\frac{1}{2}$	--	0.08	12.2	Darlot	Seed	26	110	1	None	341.3
9	J. W. Moffat	Liegan	8 0 29	N. 40 9 45	4050	2 N.	1 45	‡ 3	18	0.17	5.8	Darlot	Seed	26	12	5	None	339.5

* Reflecting telescope.
† Objective on the Stokes-Pickering plan, with reversible crown lens.
‡ Used 1-inch diaphragm.
§ Objective not corrected for photographic rays, and stopped down to $1\frac{3}{4}$ inches.
¶ Details of development were not furnished.
‖ Refigured by Clark & Sons.

Plate I.

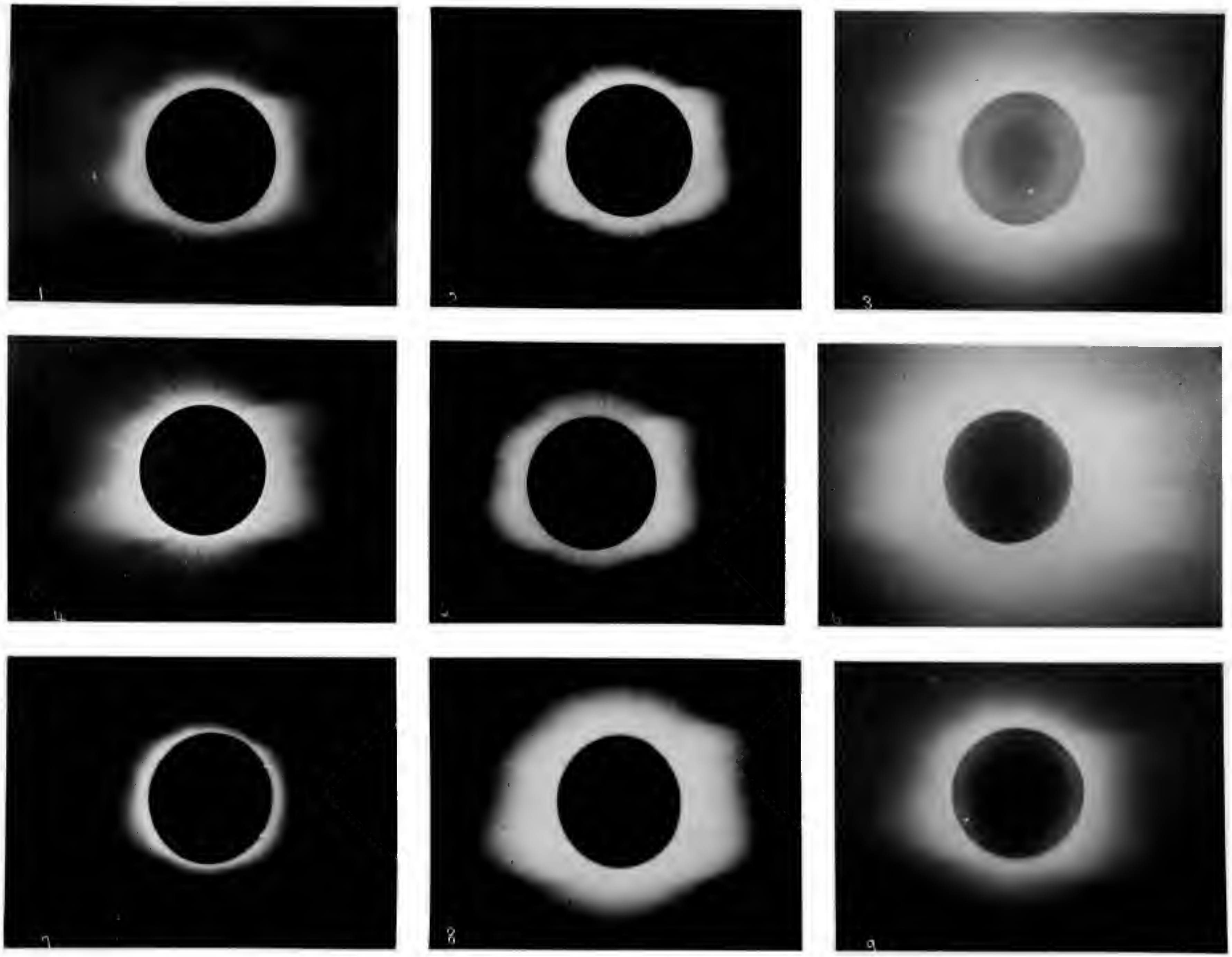
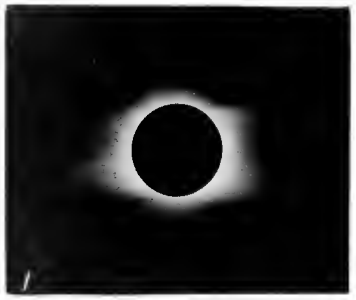
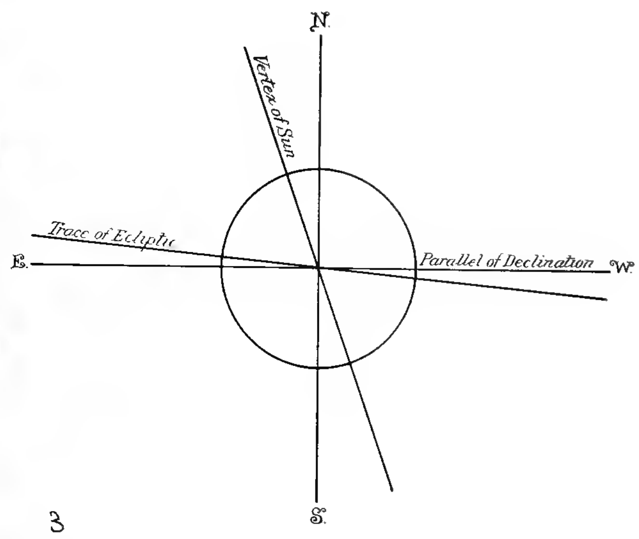


PLATE II.



1. BARNARD'S PHOTOGRAPH, No. 4.
(Size of the original.)

2. PICKERING'S PHOTOGRAPH, No. 5.
(Size of the original.)



3. DIAGRAM SHOWING VERTEX OF THE SUN AND TRACE OF THE ECLIPTIC.

p 68805

THE SOLAR CORONA,

DISCUSSED BY SPHERICAL HARMONICS.

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ADVERTISEMENT.

The following mathematical study of the solar corona, as shown in the photographs taken by Messrs. Pickering and Barnard during the total eclipse of January 1, 1889, is submitted to astronomers and physicists as a possible clue to the explanation of the corona, and as suggesting the direction to be taken in future observations and investigations.

The paper has been recommended for publication by Professors Asaph Hall and Cleveland Abbe, to whom it was referred in accordance with the usage of the Smithsonian Institution.

S. P. LANGLEY,
Secretary.

SMITHSONIAN INSTITUTION,
WASHINGTON. *October* 1, 1889.

THE SOLAR CORONA,

DISCUSSED BY SPHERICAL HARMONICS.

By Professor FRANK H. BIGELOW.

The difficulty of analyzing the structure of the solar corona is increased by the superposition of individual rays in projection on a plane perpendicular to the line of sight. The polar streamers and the outline of the equatorial wings are relatively free from this overlapping, and the body of the moon, in transit, cuts off such rays over the disk as are most distorted, so that the problem ought to be soluble by some theory applicable to the case of the rays specified

The structure to be accounted for consists: (1) of polar rays nearly vertical at the coronal poles or axis of reference for the symmetrical figure, but inclining more from this axis than a radius vector to any point as the vectoral angle increases; (2) four wings disposed upon two axes, each inclined at an angle of about 40° from the vertical; and (3) extensive equatorial wings seen more distinctly at periods of solar quiescence. This appearance upon a meridian section must be translated into corresponding zones and sectors on the figure of revolution of the sun.

We propose to treat this subject by the theory of spherical harmonics, on the supposition that we see a phenomenon similar to that of free electricity, the rays being lines of force and the coronal matter being discharged from the body of the sun, or arranged and controlled by these forces. In order to give the solution a general foundation the important points of the theory of harmonics specially relating to the case will be recapitulated, and the corresponding geometrical solution will be given in a notation adapted to the sun. My references are to Maxwell, Mascart and Joubert, Ferrer, Todhunter, Thomson and Tait in their treatises on harmonics.

THE HARMONIC THEORY.

Assume the centre of the sun's corona as the origin, the co-ordinate axes, X, Y, Z, at any instant being the radius vector to the observer, that at right angles, and the polar axis respectively. Take any set of secondary polar axes distributed at will over the spherical surface, each axis, h_1 h_2, etc., being defined as a definite direction from the origin, the cosines of the angles between these axes being $\cos m_{1.2}$, $\cos m_{1\,3}$, etc., in all combinations. Let any point in space be defined as $(r, \theta)_1$ from axis h_1, $(r, \theta)_2$ from axis h_2, etc. Then suppose there are n axes, and that s is the number of cosines between them. Assume that σ is the number of poles of the n axes distributed uniformly on the equator from X at distances $\frac{\pi}{\sigma}$. From the point (r, θ) draw planes perpendicular one to each axis and successively differentiate the equation $V = \frac{M}{r}$ relatively to each pole.

It is known that La Place's equation, $\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0$, is satisfied by a solid harmonic of the degree i of the form $H_i = \lfloor\underline{i}\, M_i\, r^i\, Y_i = r^{2i+1}\, V_i$.

In order that for a spherical closed surface the potential may satisfy the equation continuously without becoming infinite at the origin or at infinity it is converted into three terms:

$$
\begin{array}{ll}
H_i = \lfloor\underline{i}\, M_i\, r^i\, Y_i & \text{within the sphere,} \\
\sigma = C\, Y_i & \text{on the spherical surface,} \\
V_i = \dfrac{\lfloor\underline{i}\, M_i\, Y_i}{r^{i+1}} & \text{without the sphere.}
\end{array}
$$

These become, when expressed in terms of C:

$$H_i = \frac{4\,\pi\,C}{2i+1} \cdot \frac{r^i}{R^{i-1}}\, Y_i\ ,$$

$$V_i = \frac{4\,\pi\,C}{2i+1} \cdot \frac{R^{i+2}}{r^{i+1}}\, Y_i\ ,$$

wherein C is a constant and Y_i is a surface harmonic. Y_i when expressed in the general trigonometrical form is:

$$Y_i = \text{sum}\left\{(-1)^s \frac{\lfloor\underline{2i-2s}}{2^{i-s}\,\lfloor\underline{i}\,\lfloor\underline{i-s}}\ \Sigma\left(\cos\theta^{i-2s}\ \cos m^s\right)\right\}.$$

In case $s = o$, and consequently no poles are assumed symmetrically disposed around the equator nor at random over the surface, but all are collected into one pole, the surface harmonic becomes a zonal harmonic, whose form is:

$$Q_i = \text{sum}_n \left\{(-1)^n \frac{\lfloor\underline{i}}{2^{2n}\,\lfloor\underline{n}\,\lfloor\underline{n}\,\lfloor\underline{i-2n}} \left(\cos\theta^{i-2n} \sin\theta^{2n}\right)\right\},$$

where n receives the values 0, 1, 2, 3, etc., for summation.

It is obvious, from the inspection of the symmetrical disposition of the corona, that we deal with only one axis, and that therefore our harmonics are of the first degree, $i = 1$. Hence:

$$
\begin{array}{lll}
Y_1 = Q_1 = 1 \ . \ \cos\theta \,, & & \\
H_1 = \dfrac{4\,\Pi\,C}{3} \ . \ r\cos\theta \,, & & \text{Inside sphere.} \\
\sigma = C \ . \ \cos\theta \,, & & \text{Upon} \quad \text{“} \\
V_1 = \dfrac{4\,\Pi\,C}{3} R^3 \ . \ \dfrac{\cos\theta}{r^2} \,, & & \text{Outside} \quad \text{“}
\end{array}
$$

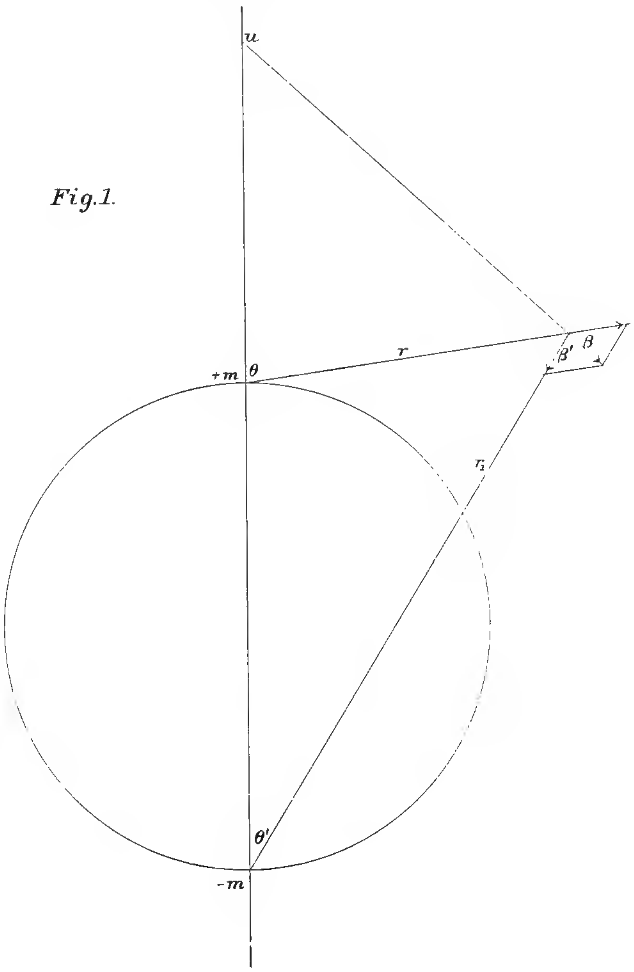
which upon differentiation satisfies the equation:

$$\frac{d\,H_1}{d\,r} - \frac{d\,V_1}{d\,r} + 4\,\Pi\,\sigma = 0.$$

THE GEOMETRICAL THEORY.

Let us now pass to the corresponding geometrical conditions. If we suppose equal masses of potential of opposite signs, $+\,m$ and $-\,m$, to be located on the extremities of the polar axis, the moment is $M = 2\,R\,m = \frac{4}{3}\,\Pi\,R^3\,C$. The equation for equipotential surfaces is $V = M\left(\frac{1}{r} - \frac{1}{r_1}\right)$ where r and r_1 are the distances from any point to the positive and negative poles respectively. (Fig. 1.)

Fig.1.



The equation of the lines of force is $(\cos\theta - \cos\theta^1) = 1 - \cos u = \frac{N}{2\pi M}$ where θ and θ^1 are the angles between r and r_1 and the positive direction of the axis, and u is the angle between the tangent to the line of force at the given point and the polar axis. If β and β^1 are the angles that this tangent makes with r and r_1 then $\frac{\sin\beta}{r^2} = \frac{\sin\beta^1}{r_1^{\,2}}$, and the force itself is $F = M\left[\frac{\cos\beta}{r^2} + \frac{\cos\beta^1}{r_1^{\,2}}\right]$.

If we suppose these potentials to be distributed over the surrounding hemispheres by the law of cosines, the surface density at any point is $\sigma = \frac{F}{4\pi}$; and if these two masses instead of being distributed are brought infinitely near together at the centre of the sphere, so that $\theta = \theta^1$ and $r = r_1$, then the equation of an equipotential surface is $V = M \cdot \frac{\cos\theta}{r^2}$, and the equation of a line of force is

$$N = 2\pi M\,\frac{\sin^2\theta}{r}, \quad \text{or} \quad (1 - \cos u) = \frac{N \cdot 2\pi \sin^2\theta}{r},$$

where N is the order of the line taken, being proportional to the square root of natural numbers. (Fig. 2.)

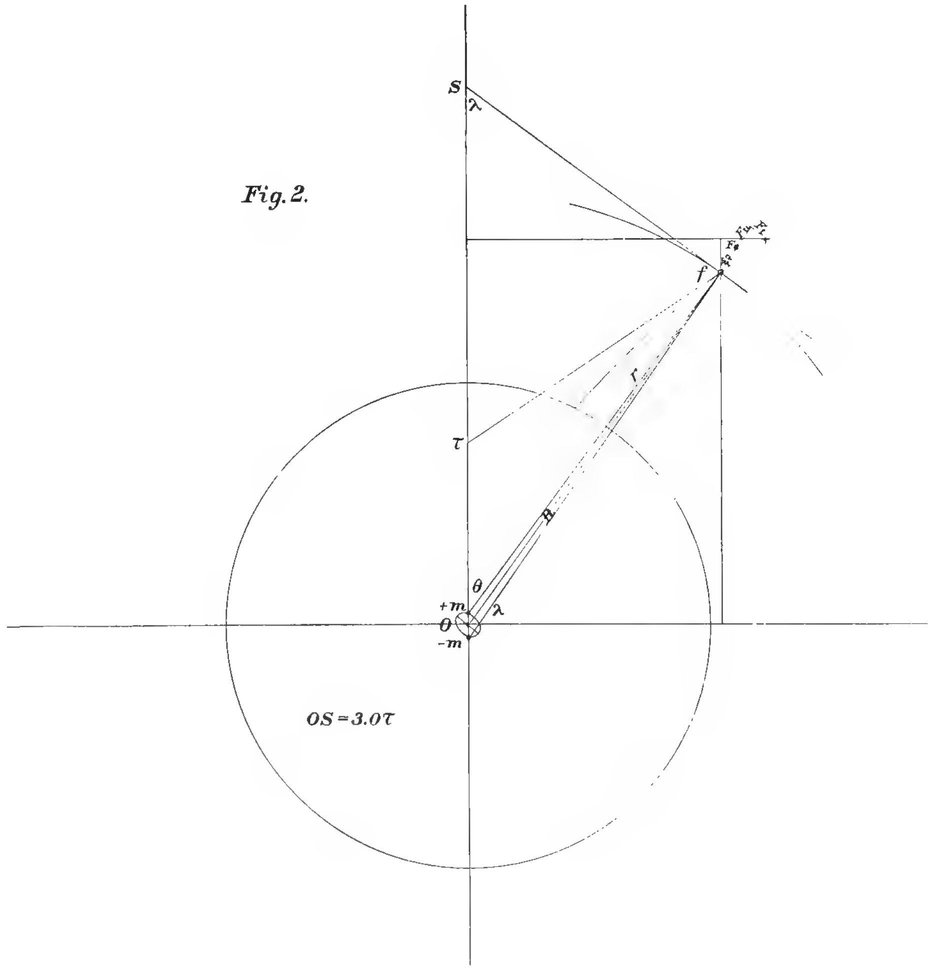


Fig. 2.

To obtain the interior force, F_i, let $r \cos\theta = 1$ in the expression for H_1; hence $F_i = -\frac{4}{3}\,\Pi\,C$, being directed opposite to the positive direction.

The moment, $M = +\frac{4}{3}\,\Pi\,C\,.\,r\cos\theta$

The interior potential, $H_1 = +\frac{4}{3}\,\Pi\,C\,.\,r\cos\theta$

The exterior potential, $V_1 = +\frac{4}{3}\,\Pi\,C\,.\,R^3\,.\,\frac{\cos\theta}{r^2}.$

Resolving the exterior potential tangentially and normally to the circle whose radius is r:

The tangential component, $F_t = +\frac{4}{3}\,\Pi\,C\,.\,R^3\,.\,\frac{\sin\theta}{r^3}.$

The normal component, $F_n = +\frac{8}{3}\,\Pi\,C\,.\,R^3\,.\,\frac{\cos\theta}{r^3}.$

Also resolving along the polar and equator axes:

The polar component, $F_p = -\frac{4}{3}\,\Pi\,C\,.\,R^3\,\frac{(1-3\cos^2\theta)}{r^3}.$

The equator component, $F_e = +\frac{4}{3}\,\Pi\,C\,.\,R^3\,\frac{3\sin\theta\cos\theta}{r^3}.$

The whole mass of potential is $m = \Pi\,R^2\,.\,C$, and the total flow of force, or the quantity $Q = (4\,\Pi\,R^2)\,\Pi\,C$.

Now construct a diagram convenient for our purpose, in reference to the corona, representing lines of equipotential and of force.

In $H_1 = \frac{4}{3}\,\Pi\,C\,.\,r\cos\theta$, and $V_1 = \frac{4}{3}\,\Pi\,C\,R^3\,\frac{\cos\theta}{r^2}$, we may regard the constants as unity; hence $H_1 = r\cos\theta$ and $V_1 = \frac{\cos\theta}{r^2}.$ In the interior of the sphere the equipotential lines are parallel to the equator. Draw a spherical meridian and divide the vertical radius into ten equal parts, each of which will represent an equal diminution of potential in passing from the maximum at the poles to zero at the equator. Outside the sphere compute $r = \sqrt{\frac{\cos\theta}{V_1}}.$ Conveniently we assume for V_1 the successive values 1.0, .9, .8, .7, .6, — — 0. and for $\cos\theta$ the same in succession. A double-entry table will give us the values of r at the angles θ corresponding to the potential V_1.

Plotting points on radii extended through the angles of equal difference of cosines and connecting all points for same V_1, we have a diagram of ovals surrounding the poles becoming tangent to the equator at the centre of the sphere.

(A table of Equipotential Surfaces is given on the following page.)

2

EQUIPOTENTIAL SURFACES.

cos θ.	log. cos θ.	θ
1.00	0.00000	0° 0′
.99	9.99564	8 7
.98	9.99123	11 29
.96	9.98227	16 16
.94	9.97313	19 57
.92	9.96379	23 5
.90	9.95424	25 51
.85	9.92942	31 47
.80	9.90309	36 52
.75	9.87506	41 25
.70	9.84510	45 34
.60	9.77815	53 8
.50	9.69897	60 0
.40	9.60206	66 25
.30	9.47712	72 33
.20	9.30103	78 28
.10	9.00000	84 16
.00	-------	90° 0′

log. V.	V.	0° 0′	8° 7′	11° 29′	16° 16′	19° 57′	23° 5′	25° 51′	31° 47′	36° 52′	41° 25′	45° 34′	53° 8′	60° 0′	66° 25′	72° 33′	78° 28′	84° 16′	90° 0′
0.00000	1.00	1.000																	
9.95424	.90	1.054	1.049	1.044	1.033	1.022	1.011	1.000											
9.90309	.80	1.118	1.112	1.107	1.095	1.084	1.072	1.061	1.031	1.000									
9.84510	.70	1.195	1.189	1.183	1.171	1.159	1.146	1.134	1.102	1.069	1.035	1.000							
9.77815	.60	1.291	1.285	1.277	1.265	1.252	1.239	1.225	1.190	1.155	1.118	1.080	1.000						
9.69897	.50	1.414	1.407	1.400	1.386	1.371	1.357	1.342	1.304	1.265	1.222	1.183	1.095	1.000					
9.60206	.40	1.581	1.573	1.565	1.549	1.533	1.517	1.500	1.458	1.414	1.369	1.323	1.225	1.118	1.000				
9.47712	.30	1.826	1.817	1.807	1.789	1.770	1.751	1.732	1.683	1 633	1.581	1.528	1.414	1.290	1.155	1.000			
9.30103	.20	2.236	2.225	2.214	2.191	2.168	2.145	2.121	2.062	2.000	1.936	1.871	1.732	1.581	1.414	1.225	1.000		
9.00000	.10	3.162	3.146	3.130	3.098	3.066	3.033	3.000	2.916	2.828	2.739	2.646	2.450	2.236	2.000	1.732	1.414	1.000	

$$r = \sqrt{\frac{\cos\theta}{V.}}.$$

The lines of force are constructed by the equation:

$$N = 2\,\Pi\left(+\,\frac{4}{3}\,\Pi\,C\,R^3\right)\frac{\sin^2\theta}{r},$$

or, calling the constant $\Pi\,R^3\,C$ unity,

$$N = \frac{\sin^2\theta}{r}\cdot\frac{8\,\Pi}{3}.$$

The successive integral numbers may be given to N, 0 . 1 . 2 3, etc., and the corresponding values of θ computed. They are:

$N = 0$	$\theta = 0$	$N = 0$	$\theta = 0$
1	20° 12′.7	5	50° 35.0
2	29 15.0	6	57 48.6
3	36 4.5	7	66 4.7
4	43 42.5	8	77 44.7

when we assume in the formula that $r = 1$. These give us the points at which the lines of force of integral orders depart from the surface of the sphere. But more conveniently for our purposes we may assign values to the angle θ, such that the cosine of the successive angles differ by one-tenth radius, and compute the values of N under this case:

If θ = 25° 51′	N = 1.593	If θ = 72 33	N = 7.624
36 52	3.016	78 28	8.043
45 34	4.272	84 15	8.294
53 8	5.362	87 8	8.357
60 0	6.283	90 0	8.378
66 25	7.037		

To trace out the path of a line of force of any order N, take $\sin^2\theta = \frac{3\,r\,N}{8\,\Pi}$, assume the required N, assign successive values to r at convenient distances, and compute θ; *e. g.*:

If N = 1.593 and r = 1	θ = 25.51	If N = 1.593 and r = 4	θ = 60.42
2	38. 4	5	77.10
3	49. 3	6	——

or assign values to θ and compute r.

(A table of Lines of Force is given on the following page.)

To find where the lines of any order N cut the equator axis, take

$$\frac{3\,r\,N}{8\,\Pi} = 1 \text{ or } r = \frac{8\,\Pi}{3\,N}.$$

assign the values to N and compute r.

N = 1.539	r = 5.248	N = 7.624	r = 1.099
3.016	2.779	8.043	1.042
4.272	1.961	8.294	1.010
5.362	1.562	8.357	1.002
6.283	1.333	8.378	1.000
7.037	1.191		

LINES OF FORCE.

$\sin^2\theta$.	$N = \frac{8\pi}{3} \cdot \sin^2\theta$.	N.	0° 0′	8° 7′	11° 29′	16° 16′	19° 57′	23° 5′	25° 51′	31° 47′	36° 52′	41° 25′	45° 34′	53° 8′	60° 0′	66° 25′	72° 33′	78° 28′	84° 16′	90° 0′
———	———	0.000	1.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
8.29960	9.22272	0.167	———	1.000	1.988	3.936	5.840	7.711	9.537	13.916	18.057	21.953	25.579	32.107	37.624	42.135	45.654	48.159	49.663	50.165
8.59806	9.52118	0.332	———	———	1.000	1.980	2.937	3.878	4.797	7.000	9.082	11.042	12 865	16.149	18 924	21.192	22.963	24.222	24.980	25.231
8.89466	9.81778	0.657	———	———	———	1.000	1.484	1.959	2.423	3.536	4.588	5.577	6.499	8.157	9.559	10.705	11.599	12.235	12 618	12.745
9.06602	9.98914	0.975	———	———	———	———	1.000	1.320	1.633	2 383	3.092	3.759	4.380	5.498	6.442	7.215	7.817	8.246	8.504	8.590
9.18672	0.10984	1.288	———	———	———	———	———	1.000	1.237	1.805	2.342	2.847	3.317	4.164	4.879	5.464	5.934	6.245	6.440	6.505
9.27900	0.20212	1.593	———	———	———	———	———	———	1.000	1.459	1.893	2.302	2.682	3.367	3.945	4.418	4.787	5.050	5.208	5.260
9.44314	0.36626	2.324	———	———	———	———	———	———	———	1.000	1.298	1.577	1.838	2.307	2.703	3.028	3.280	3.460	3.569	3.605
9.55624	0.47936	3.015	———	———	———	———	———	———	———	———	1.000	1.216	1.417	1.778	2.084	2.333	2.528	2.667	2.750	2.778
9.64110	0.56422	3.666	———	———	———	———	———	———	———	———	———	1.000	1.165	1.463	1.714	1.919	2.080	2.194	2.262	2.285
9.70748	0.63060	4.272	———	———	———	———	———	———	———	———	———	———	1.000	1.255	1.471	1.647	1.785	1.883	1.941	1.961
9.80622	0.72934	5.362	———	———	———	———	———	———	———	———	———	———	———	1.000	1.172	1.312	1.422	1.500	1.547	1.562
9.87506	0.79818	6.283	———	———	———	———	———	———	———	———	———	———	———	———	1.000	1.120	1.214	1.280	1.320	1.333
9.92424	0.84736	7.037	———	———	———	———	———	———	———	———	———	———	———	———	———	1.000	1.083	1.143	1.179	1.191
9.95908	0.88220	7.624	———	———	———	———	———	———	———	———	———	———	———	———	———	———	1.000	1.055	1.088	1.099
9.98228	0.90540	8.043	———	———	———	———	———	———	———	———	———	———	———	———	———	———	———	1.000	1.031	1.042
9.99564	0.91876	8.294	———	———	———	———	———	———	———	———	———	———	———	———	———	———	———	———	1.000	1.010
0.00000	0.92312	8.378	———	———	———	———	———	———	———	———	———	———	———	———	———	———	———	———	———	1.000

$N = \frac{8\pi}{3} \sin^2\theta$. $r = \frac{8\pi}{3} \frac{\sin^2\theta}{N}$. $\log. \frac{8\pi}{3}$ 0.92312.

To find the order of line of force at the earth's mean distance from the sun, take the mean semi-diameter of the sun, 962″, and the mean parallax of the earth, 8″.848, and the earth is 108.7 radii of the sun distant from it. For $r = 109$, $N = 0.07686$. The angular distance from the pole at which this line of force leaves the sun is 5° 29′ 47″.

Graphically the lines of force cut the equipotential lines orthogonally, and may be so drawn, starting at the points of the surface heretofore marked by the equipotentials. These lines are ovals cutting the equator perpendicularly and becoming tangent to the polar axis at the centre of the sphere. A test of the accuracy of the drawing is found by taking the sides of any of the quadrilateral figures, wherein the ratio of the mean distance between consecutive equipotential surfaces is to the mean distance between consecutive lines of force as the half the distance of the centre of the figure from the polar axis is to the unit of measure.

APPLICATION TO THE CORONA.

An analysis of these lines of force appears to be a description of the visible solar corona, and this analogy first suggested the explanation of the phenomena now given. The concentration of potential at each pole throws lines vertical at the polar region, bending gradually each side, and at a distance of 26° losing one-tenth of the force, — the angle of the line of force to the polar axis bring nearly 45°; this curve closes on the equator at 5.25 radii from the centre. The next decimal line leaves the sphere at an angle of 67° to the vertical axis, and having a potential of eight-tenths closes on the equator at 2.8 radii. The third line of force is inclined at 76° to the axis, and having potential seven-tenths closes on equator at 1.96 radii. The fourth line starts perpendicular to the vertical axis, leaves the sun at polar distance 53°, closing on equator at 1.56 radii. The other lines rapidly become more nearly parallel with the surface and close in as they lose potential.

The solar corona can now be analyzed. The straight polar rays of high tension carry the lightest substances, as hydrogen, meteoric matter, débris of comets, and other coronal material, away from the sun, and they become soon invisible by dispersion. Next we come to the strong quadrilateral rays of potential .9 .8 .7 .6, which united form the appendages conspicuously seen at periods of great solar activity. They rapidly diminish in intensity, and at the distance of one radius have generally a potential of one to two tenths. The explanation of the long equatorial wings, with absence of well-marked quadrilaterals, seen at periods of minimum activity, is that they are due to the closing of the lines of force about the equator. The re-entrance of these lines forms along the equator,

the place of zero potential, a sort of pocket or receptacle wherein the coronal matter is gradually carried by the forces, accumulated and retained as a solar accompaniment. During periods of inactivity or low maximum potential the streams along the region $40^\circ - 60^\circ$ polar distance diminish in intensity, so that huge volumes are not carried away from the surface, but none the less what does leave the sun is persistently transported to the equatorial plane of the corona. In fact, the zodiacal light may be the accumulation at great distances from the sun along this equator of such like material, being carried by forces, all of which approach the equator perpendicularly, but there become zero. Here the zodiacal coronal material has no way of escape, being once deposited.

We have a test of the accuracy of our theory which may be applied to any portion of the coronal rays, using the caution that we deal with true rays undisturbed by perspective and diffraction, and notably the polar and the outer boundaries of the quadrilaterals are best available. From the centre of the sun, with a radius vector r, draw a circle at any chosen point of such ray where the curvature is well marked, and a tangent to the circle, prolonged to intercept the polar axis with which it makes an angle. (See Fig. 2.) Let f equal the angle at which the line of force crosses this tangent; draw another tangent to the line of force and prolong it to the polar axis, then $\tan f = 2 \cot \theta = 3 \tan \lambda$. The intercept cut off from the centre of the sun by the force-tangent is one-third the intercept cut of by the circle-tangent. I believe that this criterion holds good on the photographs taken during recent eclipses, as the following readings show :

(A table of Readings is given on the opposite page.)

These readings were taken from Professor Holden's diagram (Monthly Notices R. A. S., April, 1889) by centering one side of a right triangle on the sun with radius 2 rotating to the several angles θ, previously selected to mark the prominent rays, and reading the other side of the triangle on the axis extended and marked on a scale with the radius as unit. Finally an edge was laid on the local lines at $(r\,\theta)$, and the reading on the axis again taken. It must be clearly kept in mind that it is not the direction of the whole ray from its base on the sun to the point $(r\,\theta)$, but the direction tangent to the ray at the point $(r\,\theta)$. Besides the local inaccuracies there may be a slight error in placing the direction of the axes, and the readings in the S. E. quadrant suggest this presumption. Still the approximation of the ratio to 3.0 is so evident as to show the application of this theory to the solar corona and also to witness the fidelity of Professor Holden's drawing.

I have just had the pleasure of seeing one of the photographs of the inner corona and one of the outer taken by the Harvard College party January 1, 1889, and the details are shown so clearly that our theory is at once able to be

READINGS ON HOLDEN'S DIAGRAM.

	Radius $r = 2$	Angle θ.	Intercepts by— Circle-Tangent.	Force-Tangent.	Ratio.	
N. W. Quadrant	--------	5°	2.01	.65	3.09	
		16	2.07	.66	3.14	
		22	2.15	.71	3.03	
		29	2.26	.50	4.51	Local curvature too straight.
		35	2.42	.59	4.10	
		47	2.83	.85	3.32	
					3.20	Mean.
N. E. Quadrant	--------	3°	2.00	.65	3.08	
		10	2.03	.72	2.82	
		16	2.07	.73	2.83	
		18	2.09	.72	2.90	
		23	2.13	.75	2.83	
		28	2.26	.74	3.01	
		36	2.45	.77	3.18	
		53	3.32	1.07	3.10	
		64	4.62	1.35	3.45	Local curvature faulty.
		72	6.70	1.20	5.58	
					2.97	Mean.
S. W. Quadrant	--------	3°	2.00	.50	4.00	Local lines in error.
		6	2.02	.55	3.67	
		11	2.04	.58	3.53	
		19	2.11	.70	3.01	
		41	2.62	.84	3.12	
		50	3.07	1.02	3.01	
		56	3.60	1.21	2.97	
					3.03	Mean.
S. E. Quadrant	--------	6°	2.02	.61	3.32	
		12	2.04	.57	3.52	
		19	2.10	.60	3.50	Whole Quadrant may need adjustment as to its axis.
		41	2.59	.72	3.59	
		51	3.08	.90	3.42	
		56	3.60	1.20	3.00	
					3.39	Mean.

The necessity is obvious of rejecting freely such lines of force as are not natural, and the difficulty of obtaining true lines is at present great.

tested. I give a table of the measures when our rule of polar intercepts is applied to the ray structure. They can be verified by any one possessing a Pickering photograph on celluloid.

THE PICKERING PHOTOGRAPHS.

THE INNER CORONA.

Radius r.	Angle θ.	Intercept by circle-tangent	Intercept by force-tangent.	Ratio.	Intercept.	Ratio.	Intercept.	Ratio.	Intercept.	Ratio.
			N. E. Quadrant.		N. W. Quadrant.		S. W. Quadrant.		S. E. Quadrant.	
1.00	5°	1.00	0.33	3.03	0.33	3.03	0.31	3.02	0.32	3.01
	10°	1.01	0.34	2.97	0.34	2.97	0.33	3.01	0.34	2.97
	15°	1.03	0.38	2.70	0.38	2.70	0.44	2.34	0.40	2.58
	20°	1.05	0.40	2.63	0.71	1.50	0.49	2.14	0.45	2.33
	25°	1.10	0.39	2.84	0.67	1.64	0.51	2.16	0.42	2.62
	30°	1.16	0.42	2.76	0.58	2.00	0.50	2.32	0.42	2.79
	35°	1.22	0.42	2.90	0.54	2.26	0.52	2.35	0.43	2.84
	40°	1.31	0.46	2.85	0.51	2.57	0.48	2.73	0.45	2.91
	45°	1.42	0.48	2.96	0.50	2.84	0.49	2.89	0.48	2.96
	50°	1.56	0.53	2.94	0.56	2.79	0.51	3.01	0 52	3.00
	55°	1.74	0.58	3.00	0.58	3.00	0.58	3.00	0.56	3.01
	60°	2.00	0.68	2.94	----	----	----	----	----	----
				2.88		2.89		2.94		2.94

THE OUTER CORONA.

Radius r.	Angle θ.	Intercept by circle-tangent	Intercept by force-tangent.	Ratio.	Intercept.	Ratio.	Intercept.	Ratio.	Intercept.	Ratio.
1.20	5°	1.21	0.40	3.00	0.41	2.95	0.43	2.81	0.41	2.95
	10°	1.22	0.42	2.90	0.42	2.90	0.67	1.97	0.42	2.90
	15°	1.24	0.43	2.87	0.44	2.84	0.62	2.00	0.42	2.95
	20°	1.28	0.44	2.90	0.63	2.00	0.63	2.00	0.45	2.84
	25°	1.32	0.45	2.93	0.64	2.01	0.58	2.28	0.66	2.00
	30°	1.39	0.47	2.96	0.65	2.29	0.54	2.39	0.68	2.00
	35°	1.47	0.50	2.94	0.63	2.33	0.55	2.67	0.63	2.33
	40°	1.57	0.53	2.96	----	----	0.54	2.91	0.65	2.41
	45°	1.70	0.56	3.00	----	---	0.57	2.98	0.60	2.83
	50°	1.87	0.60	3.01	----	----	0.61	3.07	----	----
	55°	2.09	0.67	3.01	----	----	----	----	----	--
				2.95		2.90		2.94		2.89

Bracketed intercepts omitted in taking means; 55 readings retained; 27 show distortion.

NOTE.—It is evident that the effect of projection of the lines of spherical harmonics on a plano is to flatten them, so that the force-tangent becomes elevated at ite intercept on the polar axis. Hence the readings of this factor are too large, and the valne of the ratio too small, by an amount depending upon the error of the curves in projection.

A scale was constructed as follows to facilitate the measurement of the lines on the photographs: A positive on glass, showing fine lines on a transparent field, was made from a drawing, which consists of concentric circles, the first coinciding with the sun's disk, the others expanding by tenths of a radius to the distance of three radii; also a series of radii at five degrees apart. The polar axis was subdivided and marked in figures, and the radii were numbered. This was reduced to the size of the picture to be discussed, and the celluloid photograph being laid against the scale and backed by a plate of glass formed a transparency which, viewed against bright sky light, rendered the direction of the rays very distinct. (See Plate I.) The circle-tangent intercept readings were taken from a table of secants: the force-tangent intercepts were read from the picture by laying an ivory scale on the ray in question. Some practice and judgment were required to distinguish true and false directions, but considerable uniformity was acquired in the way of independent measures.

An inspection of the table for the inner and the outer corona shows a decided determination of the constant ratio 3.00. In the N. E. quadrant for both coronas there is hardly a divergence from it; in the N. W. quadrant between 20° and 40° there is a sharp change indicating some disturbance at this place; the S. W. quadrant shows a similar confusion; and the S. E. is again quite regular. Some solar currents seem to have swept the poles and the rays on the western side of the sun.

It may be mentioned that these readings are for individual pictures and with poles selected by best judgment. A comparison of many pictures taken at different times and with various kinds of apparatus under the assumption that our constant 3.00 is a fundamental ratio may lead to valuable deductions as to the coronal forces. In this connection the solar prominences and the fibres of the chromosphere should be compared with the direction of the lines of force as they leave the solar surface.

It is hoped that future eclipses may furnish us with pictures of the corona so clear that the measures may be made with certainty.

It is plain that the accuracy of the results depends upon our ability to locate the polar axes. The general radiation at the poles shows the approximate position, and the radial ray is probably near the vertex, but if our rule is granted as true for the corona it becomes a means of fixing the pole precisely, referred to the whole structure of a hemisphere, rather than leaving us to depend upon appearances of rays, which probably undergo a certain amount of local variation.

The assumption regarding the poles of the corona has usually been that they are in a diametral line passing through the centre of the sun. Upon applying the principle just stated to the southern vertex, at first assuming that it lay on

SOLAR CORONA.—BIGELOW. PLATE I.

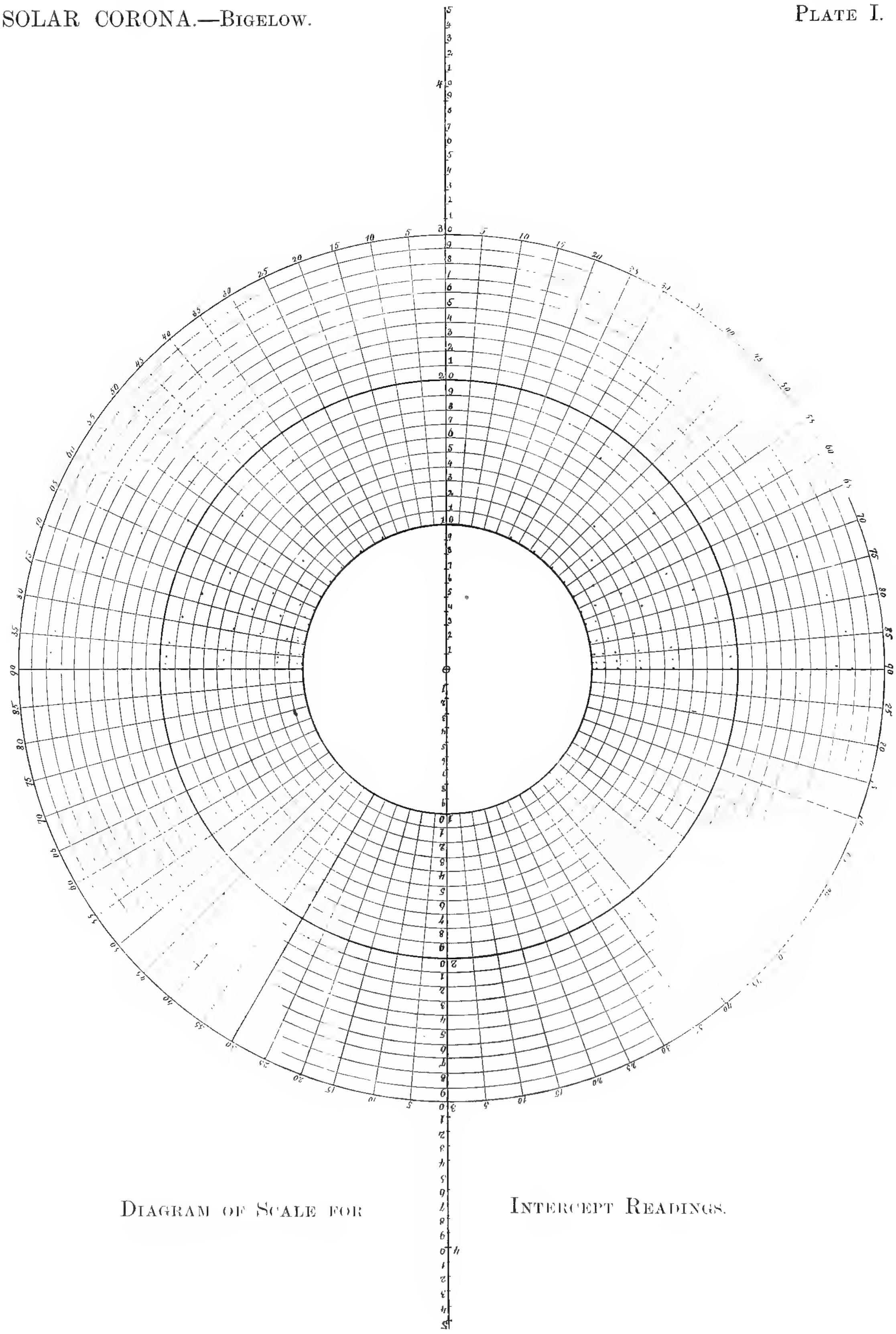


DIAGRAM OF SCALE FOR INTERCEPT READINGS.

the same diameter as the northern, I found that my intercept ratios were untrue. However, on taking the vertex in the southwestern quadrant at 169° from the northern, the readings were rectified.

We have avoided speaking of the apparent coronal structure as a phenomenon of electricity in deference to the doubt that free electricity can exist at such high temperatures as prevail on the sun's surface, but have shown that some force is present acting upon the corona according to the laws of electric potential. An inverse argument might at once be drawn from this applicability of the formulæ of statical electricity to the coronal structure that a form of energy analogous to electricity exists on the surface of the sun, but we need not insist upon the name of the active repulsive force whose potential we are discussing.

The value of the potential at any point of a line of force can be easily computed, but a diagram plotted in tenths-potentials renders the work very simple. Referring again to the Holden drawing, and for the present calling C equal to unity, we may estimate the value of the potential at the edges of the corona as recorded by the photograph.

At the north pole the rays extend to potential	0.35
At the south pole the rays extend to potential	0.50
At the northwest quadrilateral the rays extend to potential .	0.15
At the southwest quadrilateral the rays extend to potential .	0.10
At the northeast quadrilateral the rays extend to potential .	0.20
At the southeast quadrilateral the rays extend to potential .	0.15

Remembering that the smaller the potential the greater the distance seen from the edge of the sun, we note that the western quadrilaterals are generally longer, extending about one and one-third radii from the sun. They are symmetrically disposed to the poles assigned by our formula, the axis of symmetry lying 85° from the north to the west. The northeast quadrilateral is shorter for the same reason, and the tendency is to make the larger amount of matter visible in the 169° side of the axis. The diminution of matter along the axis of symmetry is very obvious. At certain parts of the quadrilateral the curvature of the rays is marked and in the right direction.

I had the diagram transferred to a transparent positive photograph scale reduced to the solar diameter, upon which the mounting was made, as above described, for the measure. It was seen at a glance, by counting the value of the lines, to what potential the matter attaching to any line of force was visible. On the same scale were produced the lines of force at the decimal potentials, and an inspection of the curvature of the computed lines and the coronal lines, when superposed, is sufficient to substantiate the truth of the theory. (See Plate II.) It is seen also that a field of accurate and intelligent study of the solar forces is now opened, and that the coronal pictures which show true structure become valuable.

SOLAR CORONA.—BIGELOW. PLATE II.

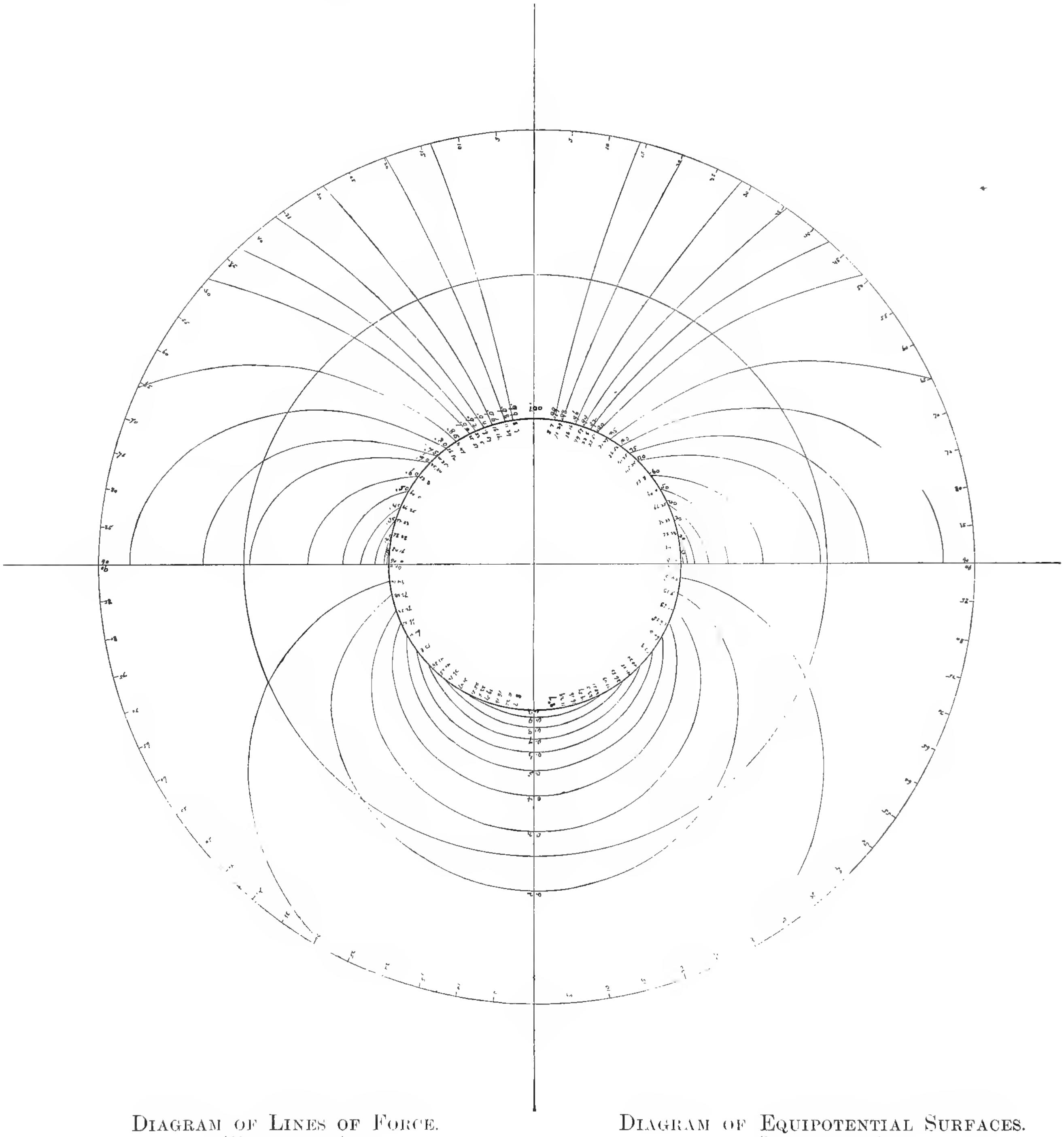


DIAGRAM OF LINES OF FORCE.
(Upper part.)

DIAGRAM OF EQUIPOTENTIAL SURFACES.
(Lower part.)

TERRESTRIAL MAGNETISM.

In treating the problem of the earth's magnetism it has been generally supposed that the forces of induction from the sun to the earth are in straight lines following the vector joining these bodies. We now see that the earth lies in a magnetic field, uniform by reason of its distance from the sun, the lines of force being directed nearly perpendicular to the plane of the earth's orbit instead of parallel to it, and of low potential, as the formula shows that the earth lies near the plane of the equator of the corona. It is not yet known exactly what relation the polar axis of the corona holds to the axis of revolution of the sun, or to the true N. and S., but it may be determined by a study of the coronal lines. If it should appear that the angle is considerable between the plane of the coronal equator and the ecliptic, even supposing the corona does not oscillate, yet the earth in its orbit must be passing through fields variable in potential and direction, which will condition some of the periodic changes of the terrestrial magnetism. Knowing the potential of the earth's magnetism and its variations the data ought to be accessible for obtaining the solar constant of maximum superficial density of electricity, and thus give a clue to the forces acting within the sun.

With the data at present available it is difficult to assign the position of the coronal pole to its true place on the solar surface, and the pictures heretofore obtained, which deal almost exclusively with the general outline of the corona instead of with the direction of the rifts and structural lines, afford little ground for deduction from symmetrical forms. If we suppose the poles of the ecliptic, the sun, and the corona to be in the plane of vision the relative places are 6°.5 from pole of ecliptic to the pole of the sun and probably 12°.5 to the pole of the corona. One rotation of the sun on its axis will then cause the coronal equator to range from about 13° north to 0° on the ecliptic. If V_1 is the maximum potential at the pole of the corona on the surface of the sun the corresponding potentials at the mean distance of the earth, 108 solar radii, are:

$$
\begin{array}{lll}
r = 108,\ \theta = 75^\circ, & V_E = 0.0000222 = & \dfrac{V_1}{45.000}. \\[2mm]
\qquad\qquad\quad 80^\circ & \qquad\quad 0.0000149 & \dfrac{V_1}{67.000}. \\[2mm]
\qquad\qquad\quad 85^\circ & \qquad\quad 0.0000075 & \dfrac{V_1}{133.000}. \\[2mm]
\qquad\qquad\quad 90^\circ & \qquad\quad 0.0000000 & \dfrac{V_1}{\infty}.
\end{array}
$$

If the corona follows the rotation of the pole the effect is to draw the lines of force up and down, north and south, in the region of the earth, so that it lies

in potentials continuously changing from zero to $\frac{V_1}{45.000}$ in a period of 26.33 days. At the same time the earth's orbital motion causes it to cut them into forces of induction, which would tend to make variations in the earth's magnetism. It is obvious that the general view is sustained that the direct magnetic influences from the sun are very slight, yet Hornstein's period should show variations confirming these coronal changes, and if the final residual of the earth's magnetic variations can be completely assorted we should have from the coronal period a means of fixing the polar positions, and also the ratio of the solar potential to that of the earth's magnetism.